PAPER SOLUTION

From Meerut

JEE MAIN

JAN	SHIFT
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Question Paper

JEE MAIN 2025 LIVE PAPER DISCUSSION

#Q. If $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $B = QPQ^T$ and matrix A is defined as A = $Q^T B^{10} Q$ (where $P = \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix}$), then trace of matrix A is

A

B

C

D

$$A = Q^T B^{10} Q = (Q^T Q) P (Q^T Q) P (Q^T \cdot) \cdots (\cdot Q) P (Q^T Q) = P^{10}$$

$$P^2 = \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & - \\ 0 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & - \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} & \\ & 1 \end{bmatrix}$$

$$P^{10} = \begin{bmatrix} 2^5 & - \\ - & 1 \end{bmatrix}$$

$$tr(A) = 32 + 1 = 33$$

Ans. (33)

#Q. Let P_i be image of parabola $P: y^2 = 4x$ with respect to line $x + y + 1 = 0$. Let the line $y + 5 = 0$ intersect P_i at A and B. If a is the distance between A and B and d be the area of triangle SAB where S is the focus of parabola P_i. Then $(a + d)$ is

(A) 8

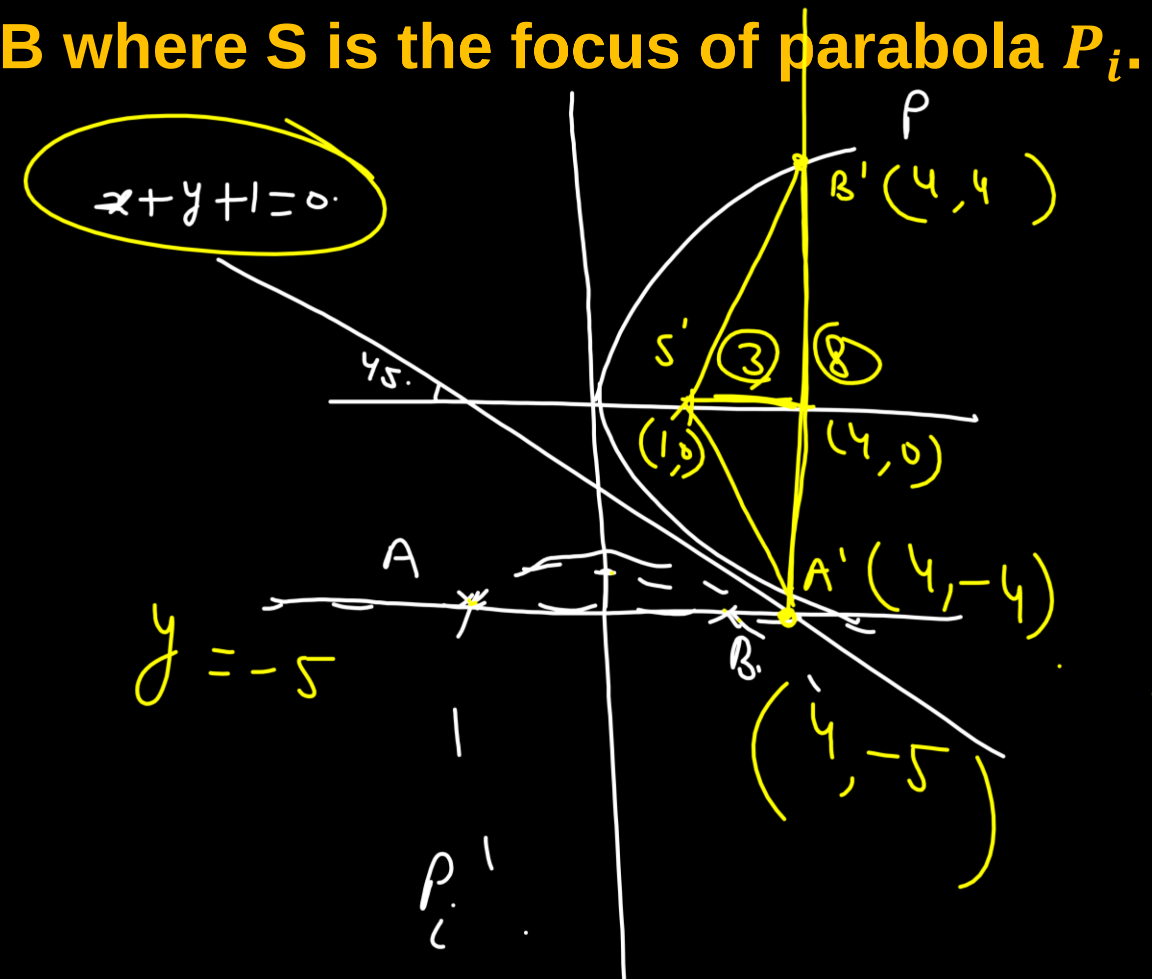
(B) 12

(C) 20 ✓

(D) 10

$a = 8$.

$d = \frac{1}{2} \times 8 \times 3 = 12$



Ans. (C)

#Q. For positive integer $n, 4a_n = n^2 + 5n + 6$ and $S_n = \sum_{K=1}^{n} \frac{1}{a_K}$. Then the value of $507(S_{2025})$ is

(A) 725

(B) 1350

(C) 540

(D) 675 ✓

$$a_k = \frac{k^2+5k+6}{4}$$

$$\frac{1}{a_k} = \frac{4}{(k+2)(k+3)} = 4\left[\frac{1}{k+2} - \frac{1}{k+3}\right]$$

$$S_n = 4\left[\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{n+2} - \frac{1}{n+3}\right]$$

$$S_n = 4\left[\frac{1}{3} - \frac{1}{n+3}\right] = \frac{4n}{3(n+3)}$$

$$507\, S_{2025} = \frac{507 \times 4 \times 2025}{3 \times 2028}$$

$$\frac{4n}{3(n+3)}$$

$$= 675$$

Ans. (D)

↑ Range

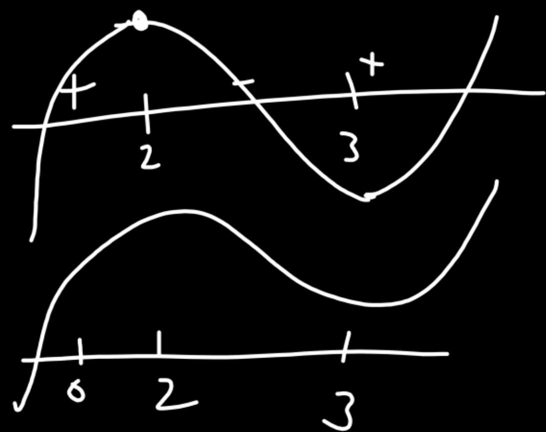
#Q. $f: [0,3] \to b$, $f(x) = 2x^3 - 15x^2 + 36x + 7$ is an onto function $g: [0,\infty) \to d$, $g(x) = \frac{x^{2025}}{x^{2025}+1}$ is also an onto function. Find the number of elements in the set $S = \{x : x \in Z, x \in b \text{ or } x \in d\}$

Sol^n

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x-2)(x-3)$



$f(2) = 2\times 8 - 15\times 4 + 36\times 2 + 7$

$= 16 - 60 + 72 + 7 = 35$

$f(0) = 7$

$b = [7, 35]$

$g(x) = \frac{x^{2025}}{x^{2025}+1}$, $0 \le x < \infty$

$\min g(x) = 0$.

Range $d = [0, 1)$. → 0 → 1

$b = [7, 35]$ → 29

30

Ans. (30)

#Q. The greatest interior angle of a polygon is 171^0 with n sides such that its angles are in Arithmetic progression with common difference of 6^0. Then n is equal to

$a = 171^\circ, \quad d = -6^\circ$

Sum of all interior angles

$(n-2)\times 180^\circ = \frac{n}{2}\left[2\times 171^\circ - (n-1)6^\circ\right]$

$180n - 360 = n\left[171 - 3n + 3\right]$

$180n - 360 = 174n - 3n^2$

$3n^2 + 6n - 360 = 0$

$n^2 + 2n - 120 = 0$

$(n+12)(n-10) = 0$

$n = -12, \ 10$ ✓

Ans. (10)

#Q. If S is a set of words formed by all the letters of word ''GARDEN'', then find the probability that vowels are not in alphabetical order.

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

Ans. (D)

#Q. In isosceles triangle two equal sides are $x + 2y = 4$, $x + y = 4$, then the sum of all possible value of slope of third side of triangle is

(A) $\frac{-2}{3}$

(B) $\frac{-3}{2}$

(C) $\frac{2}{3}$ ✓

(D) $\frac{3}{2}$

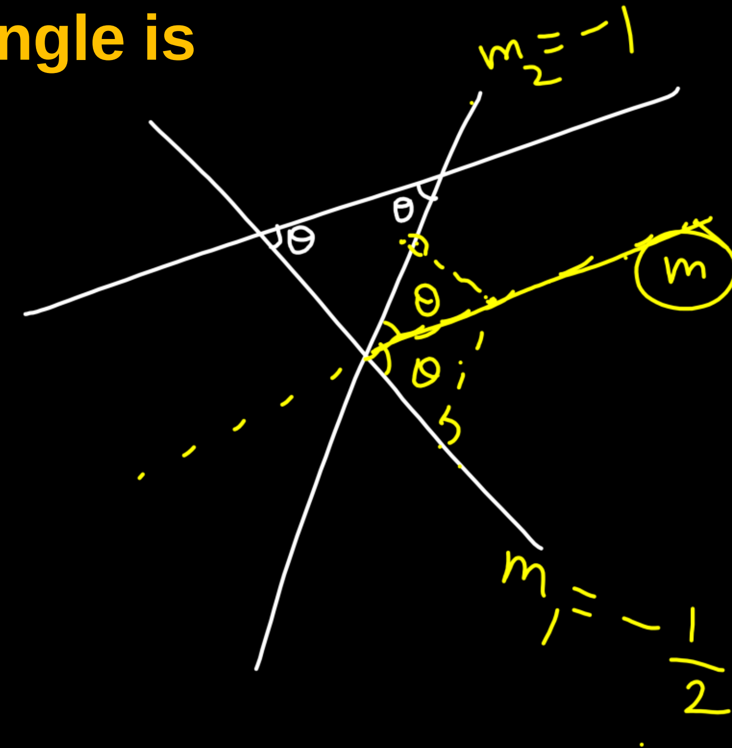
$$\tan\theta = \frac{m - \left(-\frac{1}{2}\right)}{1 + \left(-\frac{1}{2}\right)m} = \frac{-1 - m}{1 + (-1)m}$$

$$\frac{2m+1}{2-m} = \frac{m+1}{m-1}$$

$$2m^2 + m - 2m - 1 = 2m + 2 - m^2 - m$$

$$3m^2 - 2m - 3 = 0$$

Sum of roots $= \frac{-(-2)}{3} = \frac{2}{3}$.



Ans. (C)

$3-2i$ $(2-2i), 1$

#Q. If $\alpha, \beta, \gamma, \delta$ are real numbers such that $\alpha + i\beta$ and $\gamma + i\delta$ are roots of the equation $x^2 - (3-2i)x - (2i-2) = 0$ (where $i = \sqrt{-1}$), then $(\alpha\gamma + \beta\delta)$ is equal to

Roots $1, 2-2i$

$\alpha + i\beta = 1 \Rightarrow \alpha = 1, \beta = 0$

$\gamma + i\delta = 2-2i \Rightarrow \gamma = 2, \delta = -2$

$\alpha\gamma = 2, \beta\delta = 0$

(A) –6

(B) 6

(C) 2 ✓

(D) –2

Ans. (C)

#Q. The domain of the function $f(x) = sec^{-1}(2[x]+1)$ is (where $[\cdot]$ represents greatest integer function)

(A) $(-\infty, -1] \cup [0, \infty)$

(B) $(-\infty, \infty) - \{0\}$

(C) $(-\infty, -1] \cup [1, \infty)$

(D) $(-\infty, \infty)$

$2[x]+1 \geq 1$ or $2[x]+1 \leq -1$

$2[x] \geq 0$

$[x] \geq 0$

0, 1, 2, 3, ...

$2[x] \leq -2$

$[x] \leq -1$

−1, −2, −3, ...

Ans. (D)

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$p = 0$

#Q. If p is the number of possible values of r such that T_r, T_{r+1}, T_{r+2} are three terms of $(a+b)^{12}$ are in geometric progression and if q is the sum of rational terms in the expansion of $\left(3^{1/4} + 4^{1/3}\right)^{12}$, then $(p+q)$ is

(A) 250

(B) 240

(C) 238

(D) 283 ✓

$$(T_{r+1})^2 = T_r \cdot T_{r+2}$$

$$\left({}^{12}C_r\, a^{12-r}\, b^r\right)^2 = {}^{12}C_{r-1} \cdot a^{13-r}\, b^{r-1} \cdot {}^{12}C_{r+1} \cdot a^{11-r} \cdot b^{r+1}$$

$$\frac{(12)!}{r!\,(12-r)!} \cdot \frac{(12)!}{r!\,(12-r)!} = \frac{(12)!}{(r-1)!\,(13-r)!} \cdot \frac{(12)!}{(r+1)!\,(11-r)!}$$

$$r(12-r) = (13-r)(r+1)$$

$$\left(3^{\frac{1}{4}} + 4^{\frac{1}{3}}\right)^{12}$$

$$^{12}C_r \; 3^{\frac{12-r}{4}} \; 4^{\frac{r}{3}}$$

$r \to$ multiple of 3 & 4.

$r \to$ multiple of 12.

$0 \le r \le 12$, $r = 0, 12$

$$\text{Sum} = {}^{12}C_0 \cdot 3^3 + {}^{12}C_{12} \cdot 4^4$$

$$= 27 + 256$$

$$= 283$$

Ans. (D)

#Q. The no of natural number between 212 & 999 such that the sum of their digits is 15

A

B

C

D

0, 6, 9 → 4
0, 7, 8 → 4
1, 5, 9 → 4
1, 6, 8 → 4
1, 7, 7 → 2
2, 4, 9 → 6
2, 5, 8 → 6
2, 6, 7 → 6
3, 3, 9 → 3

3, 4, 8 → 6
3, 5, 7 → 6
3, 6, 6 → 3
4, 4, 7 → 3
4, 5, 6 → 6
5, 5, 5 → 1

64

Ans. (64)

#Q. There are three bag such that bag 1 has 4 white, 6 blue, bag 2 has 6 white and 4 blue and bag 3 has 5 white and 5 blue balls. A bag is randomly selected and a ball is randomly picked out of it, it comes out to be white then probability that selected bag was bag 2.

(A) $\frac{2}{5}$ ✓

(B) $\frac{2}{15}$

(C) $\frac{1}{15}$

(D) $\frac{7}{15}$

$$\frac{P(II)\,P\left(\frac{W}{II}\right)}{P(I)\,P\left(\frac{W}{I}\right)+P(II)\,P\left(\frac{W}{II}\right)+P(III)\,P\left(\frac{W}{III}\right)} = \frac{\frac{6}{10}}{\frac{4}{10}+\frac{6}{10}+\frac{5}{10}}$$

$$= \frac{6}{15} = \frac{2}{5}$$

Ans. (A)

#Q. Area bounded between the curves $C_1: x(1+y^2)-1=0$ and $C_2: y^2-2x=0$ is (in sq. unit)

A $\frac{\pi}{6}+\frac{1}{2}$

B $\frac{\pi}{2}-\frac{1}{3}$

C $\frac{\pi}{4}-\frac{1}{6}$

D $2\left(\frac{\pi}{2}-\frac{1}{6}\right)$

$1+y^2=\frac{1}{x}$

$x\to 0,\ y\to\infty$

$y=0\Rightarrow x=1$

$1+2x=\frac{1}{x}$

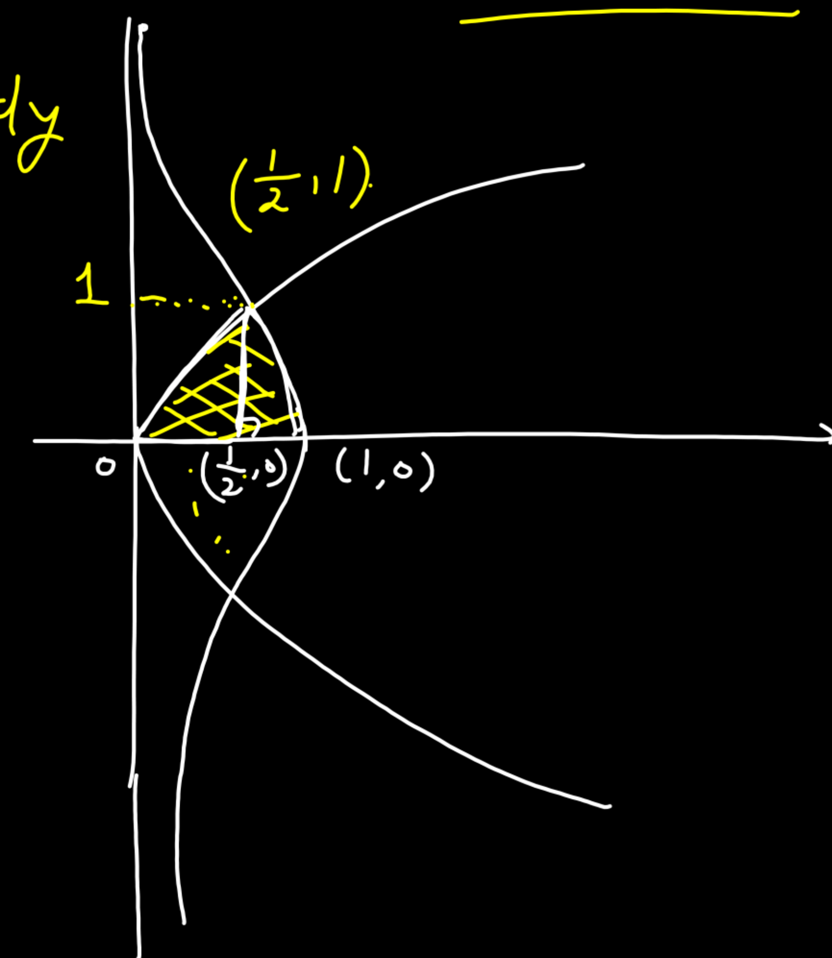
$2x^2+x-1=0$

$2x^2+2x-x-1=0$

$2x(x+1)-1(x+1)=0$

$x=\frac{1}{2},\ -1$

$2\int_0^1\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy$



$$2\int_0^1 \left(\frac{1}{1+y^2} - \frac{y^2}{2}\right) dy.$$

$$= 2\left[\tan^{-1} y - \frac{y^3}{6}\right]_0^1$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{6}\right]$$

$$= \frac{\pi}{2} - \frac{1}{3}$$

Ans. (B)

#Q. Evaluate

$$\sum_{r=1}^{13} \frac{1}{\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]\sin\left[\frac{\pi}{4}+\frac{r\pi}{6}\right]}$$

$$= 2\sum \frac{\sin\left[\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\right]}{\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$$

(A) $3\sqrt{2}-4$

$$= 2\sum \frac{\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\cos\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$$

(B) $3\sqrt{2}+2$

(C) $2\sqrt{3}-2$

$$= 2\sum_{r=1}^{13}\left[\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right]$$

(D) $2\sqrt{3}+2$

$$2\sum_{r=1}^{13}\left[\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right]$$

$$2\left[\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)+\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)-\ldots-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right]$$

$$=2\left[1-\cot\left[\left(\frac{\pi}{4}+\frac{\pi}{6}\right)+2\pi\right]\right]$$

$$=2\left[1-\cot 75^\circ\right]$$

$$=2\left[1-(2-\sqrt{3})\right]$$

$$=2(\sqrt{3}-1)$$

$$\cot 75^\circ=\tan 15^\circ$$

$$=\tan(45^\circ-30^\circ)$$

Ans. (C)

#Q. Let $f(x) = \int \frac{dx}{x^{\frac{1}{4}}(x^{1/4}+1)}$. If $f(0) = -6$ and $f(1) = a\ln 2 + b$ then $|a+b| =$

(A) 4

(B) 12

(C) –4

(D) 2

$a = 4$, $b = -8$

$x = t^4$

$dx = 4t^3\,dt$

$$\int \frac{4t^3\,dt}{t(t+1)} = 4\int \frac{t^2-1+1}{t+1}\,dt$$

$$= 4\int \left(\frac{(t-1)(t+1)}{t+1} + \frac{1}{t+1}\right)dt$$

$$= 4\left[\frac{t^2}{2} - t + \ln|t+1|\right] + c$$

$$f(x) = 4\left[\frac{x^{\frac{1}{2}}}{2} - x^{\frac{1}{4}} + \ln\left|x^{\frac{1}{4}}+1\right|\right] + c$$

$x = 0$

$-6 = c$

$$f(1) = 4\left[\frac{1}{2} - 1 + \ln 2\right] - 6$$

$$= 4\left[-\frac{1}{2} + \ln 2\right] - 6$$

$$= -8 + 4\ln 2$$

Ans. (A)

#Q. $x^2 + y^2 - 8x = 0, \frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at A, B, A triangle is formed using vertices A, B, C where C lies on $2x - 3y + 4 = 0$ find locus of centroid of ΔABC.

(A) 6x+9y-20 = 0

(B) 6x–9y+20 = 0

(C) 6x–9y–20 = 0

(D) 6x+9y+20 = 0

Ans. (C)